

**BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI**

**WORK INTEGRATED LEARNING PROGRAMMES**

**COURSE HANDOUT**

**Part A: Content Design**

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| **Course Title** | Mathematical Foundations for Data Science |
| **Course No(s)** | DSECL ZC416 |
| **Credit Units** | 4 |
| **Course Author** | G Venkiteswaran |
| **Version No** | 3 |
| **Date** | 15.04.2021 |
| **Lead Instructor** | G Venkiteswaran |

**Course Description**

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| Vector and matrix algebra, systems of linear algebraic equations and their solutions; eigenvalues, eigenvectors and diagonalization of matrices; Calculus and optimization; Counting principles and combinatorics |

**Course Objectives**

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| **No** | Objective- The course aims to |
| **CO1** | Introduce concepts in linear algebra and to use it as a platform to model physical  problems. |
| **CO2** | Provide techniques for analytical and numerical solutions of linear equations and introduce the  concept of convergence. |
| **CO3** | Utilize concepts of linear algebra and calculus in solving optimization problems. |
| **CO4** | Introduce some of the concepts in combinatorics |
| **CO5** | Introduce some concepts from counting principles |

**Text Book(s)**

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| --- | --- |
| **No** | **Author(s), Title, Edition, Publishing House** |
| T1 | Erwin Kreyszig, Advanced Engineering Mathematics, Wiley India, 10th Edition, 2015 (earlier editions are also okay) |
| T2 | Kenneth H. Rosen, Discrete Mathematics and its Applications, Tata McGraw Hill, 7th Ed., 2011. |

**Reference Book(s) & other resources**

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| --- | --- |
| **No** | **Author(s), Title, Edition, Publishing House** |
| R1 | K Hoffman and R Kunze, Linear Algebra, Pearson Education, 2nd Edition, 2005. |
| R2 | Kolman, Busby, Ross and Rehman, Discrete Mathematical Structures for Computer Science,  Pearson Education, 6th Edition, 2017 |
| R3 | Operations Research: An Introduction, Pearson Education, 10th Edition, 2017. (Earlier editions  are also okay) |

**Content Structure**

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| **No** | **Title of the module** | **References** |
| M1 | 1. Matrices, rank, determinants, solution of linear systems – analytical techniques  1.1. Solution of linear systems (A (m x n) x (n x 1) = b (m x 1); A has rank r.) – just a recapitulation  1.2. Solution using Gauss elimination with and without pivoting and operations count  1.3 LU decomposition methods  1.4 Iterative methods for linear systems | T1: Sec 7.3, 20.1, 20.2, 20.3 |
| M2 | 2. Vector spaces and subspaces, basis and dimensions, Linear transformations and properties  2.1 Vector spaces, inner product spaces, properties, LI and LD, bases and dimensions, LT  2.2. LT and Rank-Nullity theorem   * 1. NS(A), RS(A) and CS(A) – illustration | T1: Sec 7.4, 7.9 |
| M3 | 3. Eigenvalues, Eigenvectors and singular values  3.1. Eigenvalues and eigenvectors of special matrices and their properties  3.2. Eigenbases and diagonalization  3.3. Gerschgorin’s Theorem  3.4 Power Method | T1: Sec 8.1,8.7, 20.7, 20.8 |
| M4 | 4. Decomposition methods (Eigenvalue, decomposition, QR and SVD)   * 1. Gram-Schmidt Orthogonalization procedure   2. QR decomposition   3. SVD   4. Dimensionality reduction | T1: Sec 20.9  Class notes |
| M5 | 1. Linear programming problems    1. Motivation – 3 problems    2. Application of LA to Simplex method | Class notes |
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| M6 | 6. Calculus and Optimization (applications from probability theory to be used for exemplification in Calculus)  6.1 Continuous functions on closed intervals, differentiation (1d case)  6.2 Taylor series expansion  6.3 Maxima and minima  6.4 Integral properties (cdf and pdf, even and off integrands, integration by parts and so on) for 1d  7. Calculus of several variables   * 1. Review limits, continuity and differentiability (graphically and algebraically)   2. Vector calculus and some of the identities   3. Maxima and minima (unconstrained)   4. Steepest gradient method   5. Lagrange multipliers (for more number of constraints) | Class notes |
| M7 | 8. Counting principles and combinatorics   * 1. Induction principle   2. Mathematical induction   3. Strong induction   4. Recursive definition and structural induction  1. Counting Principles    1. Basics of counting    2. Pigeonhole principle    3. Permutations and combinations    4. Binomial coefficients and identities 2. Advanced counting    1. Application of recurrence relations    2. Solving linear recurrence relations    3. Generating functions | T2: Sec 5.1-5.3  T2: Sec 6.1-6.4  T2: Sec 8.1, 8.2, 8.4 |

**Learning Outcomes:**

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| No | Learning Outcomes |
| LO1 | Students will be able to effectively use matrix algebra tools to analyse and solve systems of linear equations. |
| LO2 | Students will be able to use some numerical methods to solve linear systems of equations |
| LO3 | Students would be able to use methods in linear algebra to solve linear programming problems and methods in calculus to solve non-linear optimization problems. |
| LO4 | Students will be able to use the methods of counting principles and combinatorics |

**Part B: Contact Session Plan**

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| **Academic Term** | Second Semester 2021-2022 |
| **Course Title** | Mathematical Foundations for Data Science |
| **Course No** | DSECL ZC416 |
| **Lead Instructor** | G. Venkiteswaran |

***Course Contents***

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| **Contact Hours** | **List of Topic Title** | **Text/Ref Book/external resource** |
| 1 | Matrices, rank, determinants, solution of linear systems – analytical techniques   * Solution of linear systems (A (m x n) x (n x 1) = b (m x 1); A has rank r.) – just a recapitulation * Solution using Gauss elimination with and without pivoting and operations count * LU decomposition methods | T1: Sec 7.3, 20.1, 20.2 |
| 2 | Numerical solution for linear systems   * LU decomposition methods (Continued) * Iterative methods for linear systems | T1: Sec 20.2, 20.3 |
| 3 | Vector spaces and subspaces, basis and dimensions, Linear transformations and properties   * Vector spaces, inner product spaces, properties, LI and LD, bases and dimensions, LT * LT and Rank-Nullity theorem * NS(A), RS(A) and CS(A) – illustration | T1: Sec 7.9 |
| 4 | Eigenvalues and eigenvectors   * Eigenvalues and eigenvectors of special matrices and their properties * Eigenbases and diagonalization * Gerschgorin’s Theorem * Power Method | T1: Sec 8.1,8.7, 20.7, 20.8 |
| 5-6 | Decomposition methods (Eigenvalue, decomposition, QR and SVD)   * Gram-Schmidt Orthogonalization procedure * QR decomposition * SVD * Dimensionality reduction | T1: Sec 20.9  Class notes |
| 7 -8 | Application of linear algebra in optimization. Modelling linear programming problem and the basics of Simplex algorithm and sensitivity analysis.   * Model a LPP in construction of buildings. * Model the currency arbitrage optimization problem. * Work out the graphical method of solution in the case of 2 variable case * Simplex method for simple cases * Outline how Gauss Jordan produces the inverse matrix. | Class notes |
| 9 | Properties of functions   * Continuous functions on closed intervals, differentiation (1d case) * Taylor series expansion * Maxima and minima * Integral properties (cdf and pdf, even and off integrands, integration by parts and so on) for 1d | Class notes |
| 10 | Calculus of several variables   * Review limits, continuity and differentiability (graphically and algebraically) * Vector calculus and some of the identities * Maxima and minima (unconstrained) * Steepest gradient method * Lagrange multipliers (for more number of constraints) | Class notes |
| 11-12 | Induction principle   * Recursive definition and structural induction * Mathematical induction * Strong induction | T2: Sec 5.1, 5.2, 5.3, |
| 13-14 | Counting Principles   * Basics of counting * Pigeonhole principle * Permutations and combinations * Binomial coefficients and identities | T2: Sec 6.1, 6.2, 6.3,6.4 |
| 15-16 | Advanced counting   * Application of recurrence relations * Solving linear recurrence relations * Generating functions | T2: Sec 8.1, 8.2, 8.4 |

*# The above contact hours and topics can be adapted for non-specific and specific WILP programs depending on the requirements and class interests.*

## *Lab Details*

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| **Title** | **Access URL** |
| Lab Setup Instructions | Not applicable |
| Lab Capsules | Not applicable |
| Additional References | Not applicable |

***Select Topics and Case Studies from business for experiential learning***

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| --- | --- | --- |
| **Topic No.** | **Select Topics in Syllabus for experiential learning** | **Access URL** |
| 1 | Assignment - linear algebra topics |  |
| 2 | Assignment- discrete structures topics |  |

***Evaluation Scheme***

Legend: EC = Evaluation Component

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| --- | --- | --- | --- | --- | --- |
| **No** | **Name** | **Type** | **Duration** | **Weight** | **Day, Date, Session, Time** |
| 1 | Assignment 1 | Online |  | 10% | Start date: 1/06/2021 |
| 2 | Assignment 2 | Online |  | 10% | Start date: 23/08/2021 |
| 3 | Quiz 1 | Online | \* | 5% | 11/06/2021 |
| 4 | Quiz 2 | Online | \* | 5% | 13/08/2021 |
| 5 | Quiz 3 | Online |  | 5% | 10/09/2021 |
| 6 | Mid-Semester Exam | Closed book | 90 min | 30% | 4/07/2021 FN |
| 7 | Comprehensive Exam | Open book | 150 min | 40% | 26/09/2021 FN |

***Important Information***

Syllabus for Mid-Semester Test (Closed Book): Topics in Weeks 1-8

Syllabus for Comprehensive Exam (Open Book): All topics (in sessions 1 to 16) given in plan of study

Evaluation Guidelines:

1. EC-1 consists of two Assignments and two Quizzes (best two out of the three would be taken for grading). Announcements regarding the same will be made in a timely manner.
2. For Closed Book tests: No books or reference material of any kind will be permitted. Laptops/Mobiles of any kind are not allowed. Exchange of any material is not allowed.
3. For Open Book exams: Use of prescribed and reference text books, in original (not photocopies) is permitted. Class notes/slides as reference material in filed or bound form is permitted. However, loose sheets of paper will not be allowed. Use of calculators is permitted in all exams. Laptops/Mobiles of any kind are not allowed. Exchange of any material is not allowed.
4. If a student is unable to appear for the Regular Test/Exam due to genuine exigencies, the student should follow the procedure to apply for the Make-Up Test/Exam. The genuineness of the reason for absence in the Regular Exam shall be assessed prior to giving permission to appear for the Make-up Exam. Make-Up Test/Exam will be conducted only at selected exam centres on the dates to be announced later.

It shall be the responsibility of the individual student to be regular in maintaining the self-study schedule as given in the course handout, attend the lectures, and take all the prescribed evaluation components such as Assignment/Quiz, Mid-Semester Test and Comprehensive Exam according to the evaluation scheme provided in the handout.